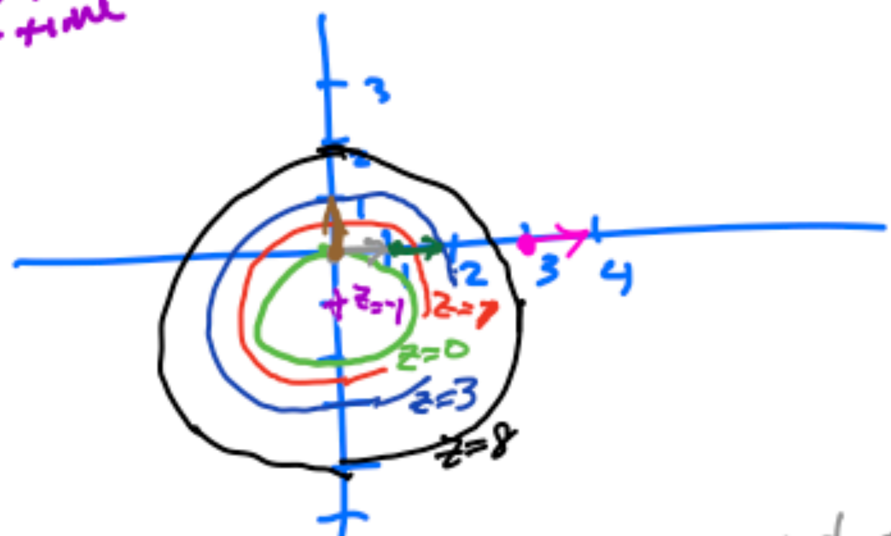
from last time

Contour Diagram for

$$f(x,y) = x^2 + y^2 + 2y = z$$



$$\frac{\partial f}{\partial x}(\overset{x}{0},\overset{y}{0}) = 2x = 0$$

rate of change of f as you move in the +x dir from (0,0)

$$\frac{\partial f}{\partial y}(0,0) = 2y + 2 = 2$$

rate of change of f as we move in the +y direction from (0,0).

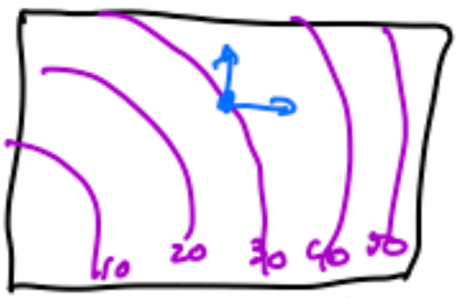
$$\frac{\partial f}{\partial x}(1,0) = 2x = 2$$

rate of change of f as we move in the x-direction from (1,0)

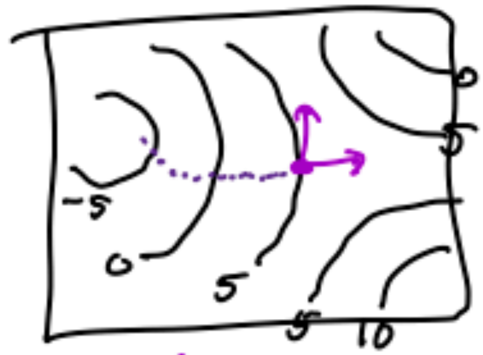
$$\frac{\partial f}{\partial x}(3,0) = 2x = 6$$

rate of change of f as we move in the x-direction from (3,0)

→ As you can see, you can guess whether the partial derivative is zero or + or − from looking at the contour diagram.

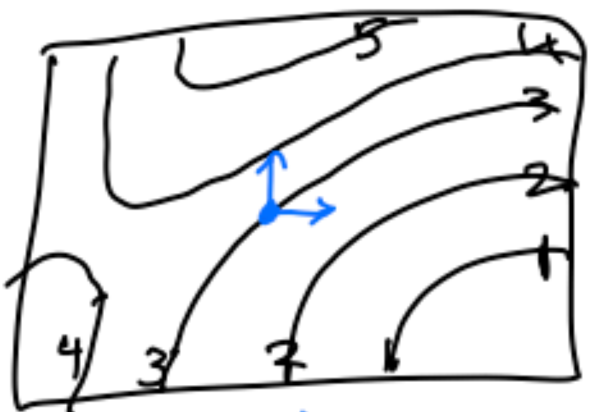


Both $\frac{\partial f}{\partial x}$ & $\frac{\partial f}{\partial y}$ are positive at the point.



At this point, it looks like $\frac{\partial f}{\partial x}$ is probably positive.

$\frac{\partial f}{\partial y}$ is 0 (direction is tangent to the contour — "level set")

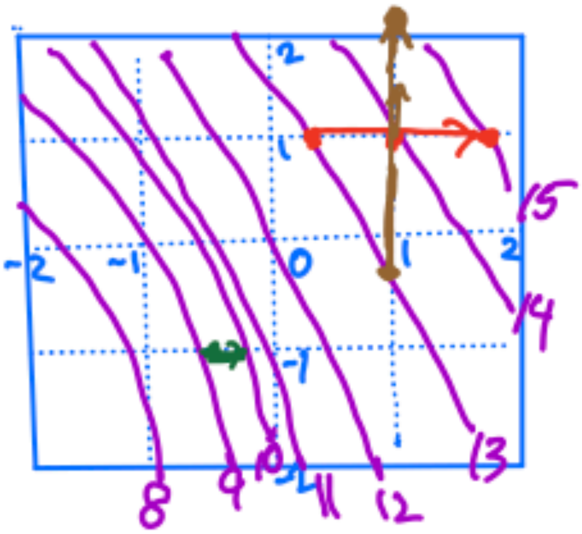


looks like:

$\frac{\partial f}{\partial x}$ is negative (at 3, going toward a value of 2)

$\frac{\partial f}{\partial y}$ is positive (at 3, going toward a value of 4)

More precise calculations:



Estimate

$$\frac{\partial f}{\partial x}(1,1) \approx \frac{\Delta z}{\Delta x} = \frac{2}{1.8-0.3} = \frac{2}{1.5} \approx 1.3$$

$$\frac{\partial f}{\partial y}(1,1) \simeq \frac{\Delta z}{\Delta y} \simeq \frac{2}{2.2-(-.3)} = \frac{2}{2.5} \simeq .8$$

$$\frac{\partial f}{\partial x}(-.5,-1) \simeq \frac{\Delta z}{\Delta x} \simeq \frac{1}{-.25-(-.6)} = \frac{1}{.35} \simeq 3.0$$

The gradient of a function of several variables.

e.g. $f(x,y) = y\,e^{xy}$

$$\nabla f = \text{grad} f = (f_x, f_y)$$

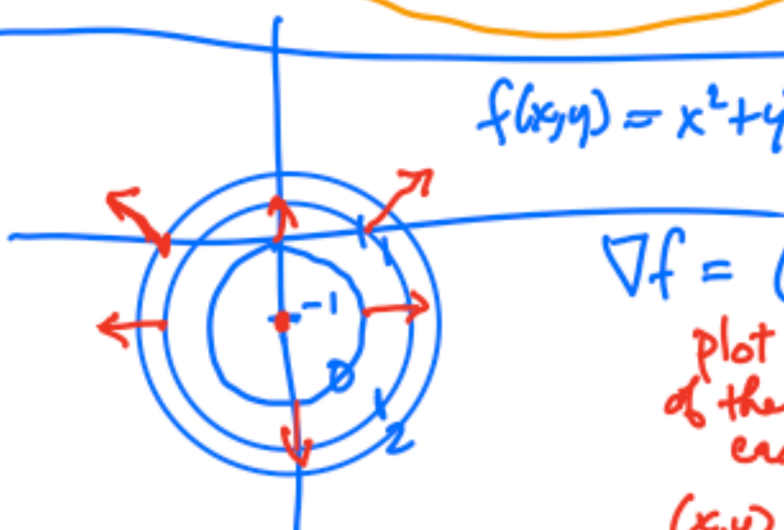
$$\nabla f = (f_x, f_y) \overset{\text{vector.}}{=} \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right).$$

$$= \left(y^2 e^{xy},\ xy e^{xy} + e^{xy} \right)$$

$$\frac{\partial}{\partial y}\left[y e^{xy} \right] = \left[y e^{5y} \right]' = (y)' e^{5y} + y (e^{5y})'$$

$$= e^{5y} + y \cdot 5 e^{5y}$$

$$= e^{xy} + yxe^{5y}$$



$$f(x,y) = x^2 + y^2 + 2y$$

$$\nabla f = (2x, 2y+2)$$

plot short versions of these vectors at each point

(x,y)	∇f
$(-1,0)$	$(-2,2)$
$(0,0)$	$(0,2)$
$(1,0)$	$(2,2)$
$(-1,-1)$	$(-2,0)$
$(0,-1)$	$(0,0)$
$(1,-1)$	$(2,0)$
$(0,-2)$	$(0,-2)$